Identifying Heavy-Hitter Flows from Sampled Flow Statistics*

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SUMMARY With the rapid increase of link speed in recent years, packet sampling has become a very attractive and scalable means in collecting flow statistics; however, it also makes inferring original flow characteristics much more difficult. In this paper, we develop techniques and schemes to identify flows with a very large number of packets (also known as heavy-hitter flows) from sampled flow statistics. Our approach follows a two-stage strategy: We first parametrically estimate the original flow length distribution from sampled flows. We then identify heavy-hitter flows with Bayes' theorem, where the flow length distribution estimated at the first stage is used as an a priori distribution. Our approach is validated and evaluated with publicly available packet traces. We show that our approach provides a very flexible framework in striking an appropriate balance between false positives and false negatives when sampling frequency is given. key words: network measurement, packet sampling, flow statistics, a priori distribution, Bayes' theorem

1. Introduction

For very high-speed links (e.g., OC-768+), collecting all packets for on-line analysis is beyond the capability of most existing measurement equipments. For example, we have only 8 ns to process a 40-byte packet at a 40 Gb/s link. Thus the demands for CPU power, memory/storage capacity and access speed to conduct network measurement are overwhelming. Being proposed as a candidate to meet this challenge, packet sampling techniques in recent years have attracted more and more attention from both industry and research communities [3], [4], [7], [12], [24]. Modern routers already had these techniques embedded, e.g., NetFlow [16] and sFlow [21]. Also, the Packet Sampling (psamp) Working Group [19] at IETF has been standardizing the techniques related to packet sampling. As being discussed in

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[2], [5], [6], [11], although packet sampling techniques provide greater scalability for network measurement, they also make inferring original flow characteristics much more difficult.

In this paper, we attempt to answer the following question: *How many* (*Y*) *sampled packets of a specific flow should be collected to conjecture that the original flow has more than X packets in total*? In particular, we are interested in identifying flows with a very large number of packets, i.e., heavy-hitter flows. Many ISPs need to regulate traffic generated by heavy-hitters according to their cumulative traffic volume over a certain time period, so a practical answer to this question is very useful in choosing an adequate length threshold of sampled flows to identify heavy-hitters. Here, *flow length* is the number of packets in a flow, *original* flow length means the number of packets in a flow that actually appear on a link during a certain time period, and *sampled* flow length means the number of sampled packets for a particular flow.

To answer more than just the above question, in this paper we develop a framework to determine the length threshold of sampled flows for identifying flows of interest. Our approach follows a two-stage strategy: We first estimate the original flow length distribution from sampled flow statistics. More specifically, we fit sampled flow statistics to a truncated Pareto distribution by means of maximum likelihood estimation (MLE). Next, we use Bayes' theorem with the estimated a priori distribution for identifying heavyhitter flows from sampled flows. As we shall see, this approach is very flexible in striking an appropriate balance between false positives and false negatives when sampling frequencies are given. Although we use the usual 5-tuple definition (i.e., source/destination IP addresses and port numbers and protocol identifier) for flows in this paper, our approach is also applicable to aggregated flows defined by their IP prefixes or AS numbers.

The remainder of the paper is organized as follows. Section 2 reviews related work and compares it with ours. Section 3 explains how to characterize the original flow length (i.e., a priori distribution) with a truncated Pareto distribution. In Sect. 4, we discuss how to estimate the original flow length distribution from sampled flow statistics by means of MLE. In Sect. 5, we describe our framework to infer the original flow length from the sampled flow length. We also validate our approach by applying extensive packet sampling processes to real packet traces. In Sect. 6, we show

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how to identify heavy-hitter flows based on our framework. Section 7 discusses some related issues. In Sect. 8, we conclude this paper with a brief summary.

2. Related Work

There are several papers addressing the problem of *inferring* original flow characteristics from sampled flow statistics. Duffield et al. [6] investigate how to estimate the original flow length distribution from sampled flow statistics. Their key idea is to use MLE for inference and additional information in measured flow records, e.g., TCP SYN flag, which can be used to estimate the number of original flows.

On the other hand, the aim of our work is to develop a way to *infer* the original flow length from the sampled flow length without looking at each packet. To do so, we need the a priori length distribution of original flows. In contrast to [6], we adopt a parametric approach to estimating the original flow length distribution, since such an approach reduces operation overhead significantly. As we shall see, our approach works well in identifying heavy-hitters. Note also that this work goes beyond our previous work [15] by introducing the above approach, i.e., the estimation of the original flow length distribution from the sampled flow statistics.

Hohn and Veitch [11] also study the related problem. Using a Poisson flow arrival model, they reveal that there exists an inherent limit in recovering the original flow length distribution from sampled flow statistics in a theoretical fashion. They also show that asymptotic properties of the heavy-tailed flow length distribution can be recovered from sampled flow statistics. Barakat et al. [2] study how to detect and rank the largest flows from sampled flow statistics. Through theoretical analysis and experiments with collected packet traces, they show that the ranking accuracy strongly depends on the sampling frequency, e.g., accurate ranking of the largest (say, the 10 heaviest) flows requires sampling rate of 10% or greater. They also show that by applying a protocol-aware ranking method, the required sampling frequency can be reduced by an order of magnitude.

There is another approach to tackling the problem of identifying heavy-hitter flows, i.e., the data reduction method. The aim of this approach is to reduce the amount of memory required for keeping flow statistics. Estan and Varghese [8] propose two novel techniques, sampleand-hold and multistage filters. Both techniques improve the process of extracting flow statistics in high-speed networks, while keeping memory consumption reasonably low. Kumar et al. [13] propose a new technique called space-code Bloom filter for extracting per-flow traffic statistics in highspeed networks. Their key ideas are the extension of the traditional Bloom filter to a one with multiple sets of hash functions and the use of multi-resolution sampling. Their approach can capture flow statistics very well, while requiring a small amount of memory resources. Golab et al. [10] propose a deterministic algorithm to identify frequent items using a memory-limited sliding window model. The proposed algorithm can fulfill its objective with limited memory resources. The main difference of these approaches from ours is their requirement of complex per-packet processing. That is, those approaches are based on non-sampling techniques, which look up all the coming packets instead of sampling them; thus it can keep rich information such as the representation of per-flow statistics. The approach has intrinsic constraints to operate with small amount of memory space (such as SRAM) to keep up with per-packet processing. It also requires to develop dedicated hardware to implement complexed operations such as hashing. In contrast, packet sampling schemes, which is our basis, have no requirement of per-packet processing and provide high scalability and low implementation cost. Actually, packet sampling is today's off-the-shelf measurement technique widely used in actual academic/commercial networks.

3. A priori Distribution of Original Flow Length

It is well known that the flow length distribution in the Internet is heavy-tailed [9], [14], [20], [22], [23], which is considered as one of the invariant characteristics of the Internet. The Pareto distribution is a simple model for characterizing heavy-tailed distributions. However, as we can see, the flow length distribution should always have an upper bound, which is dictated by the following facts. First, many Internet flows correspond to data files, and there is a limit on the file/data size in most file systems. For example, the capacity of a regular DVD disc is 4.377 GB, and the FAT32 file system supports the maximum file size of 4 GB. Also, since the measurement time period is always finite, the observed flow length should be bounded by the number of total observed packets, even if there exists only one flow. Thus, when we observe the flow length distribution in a finite time period, there is always a cutoff length that truncates the heavy-tail distribution. Here, we adopt the truncated Pareto distribution to approximate the flow length distribution. The truncation point can be estimated from the observed traffic in the following way.

We assume that there are *m* flows observed, i.e., without sampling, during a certain time period. Let X_j (j = 1, 2, ..., m) denote the original flow length of the *j*-th flow, where a flow is defined by its 5-tuple identity throughout this paper. We then assume that the distribution of X_j 's can be approximated by a *discrete*, *truncated* Pareto distribution:

$$\Pr[X_j = x] = \frac{x^{-\theta} - (x+1)^{-\theta}}{1 - (\nu+1)^{-\theta}} \qquad (1 \le x \le \nu),$$

$$\Pr[X_j > x] = \frac{(x+1)^{-\theta} - (\nu+1)^{-\theta}}{1 - (\nu+1)^{-\theta}} \qquad (0 \le x \le \nu).$$

Note that X_j has its upper bound ν (i.e., $\Pr[X_j > \nu] = 0$), and every flow contains at least one packet (i.e., $\Pr[X_j \ge 1] = 1$). This truncated distribution is a modified version of the original truncated Pareto distribution studied in [1]. Note also that X_j is a discrete variable for flow length.

Given the observed flow length X_j (j = 1, 2, ..., m), parameters ν and θ can be estimated by means of MLE. The



Fig.1 A priori distribution estimated by the truncated Pareto distribution.

ML estimator \hat{v} of v is given by

$$\hat{\nu} = \max\left(X_1, X_2, \dots, X_m\right),\tag{1}$$

and the ML estimator $\hat{\theta}$ of θ satisfies the following equation:

$$-m\frac{\log(\hat{v}+1)(\hat{v}+1)^{-\hat{\theta}}}{1-(\hat{v}+1)^{-\hat{\theta}}} + \sum_{j=1}^{m}\frac{-\left(\log X_{j}\right)X_{j}^{-\hat{\theta}} + \left\{\log\left(X_{j}+1\right)\right\}\left(X_{j}+1\right)^{-\hat{\theta}}}{X_{j}^{-\hat{\theta}}-\left(X_{j}+1\right)^{-\hat{\theta}}} = 0.$$
(2)

The proofs of Eqs. (1) and (2) are given in Appendix A.

To evaluate the accuracy of the truncated Pareto approximation, we use a packet trace collected at an OC-48c backbone link by the PMA project at NLANR [17]. The trace was collected from 10:50 to 11:00 on August 14, 2002. For easy handling, we use the first 10^7 packets that correspond to about 124 seconds of observed traffic. We refer to this sliced packet trace as trace-I throughout this paper. The total number of flows in trace-I is 206, 299.

Figure 1 shows the Log-Log-Complementary-Distribution (LLCD) plots of the empirical distribution of the original flow lengths (circles) and the truncated Pareto approximation for trace-I (curve). We observe that the truncated Pareto model fits the empirical distribution fairly well. Note that a rapidly decreasing tail is one of typical features in the truncated Pareto distribution.

In Fig. 2, we show the corresponding quantilesquantiles (Q-Q) plot to further validate the accuracy of the truncated Pareto approximation. The horizontal axis represents the empirical quantiles, and the vertical axis represents the quantiles of the approximate truncated Pareto distribution. We observe that the curve is fairly close to a straight line with slope 1 over a wide quantile range, although there are deviations in the middle of the support of the distribution.



Fig. 2 Q-Q plot for the flow length distribution: empirical distribution vs. approximate distribution.

 Table 1
 Estimated parameters in the truncated Pareto distribution.

Ŷ	$\hat{\theta}$
244, 739	.891

Table 1 shows the estimated values of parameters v and θ for trace-I. It is interesting to see that the estimated shape parameter $\hat{\theta}$ is less than 1, which does not imply the infinite average of flow length. Since we adopt the truncated distribution, the estimated flow length distribution always has finite mean and variance.

Since the truncated Pareto distribution is characterized by only two parameters, it is quite easy to estimate them. Thus the model offers higher feasibility in actual operation. Of course, the truncated Pareto distribution might not be sufficient to characterize the flow length distribution in all cases. Some statistical metrics such as the *weighted mean relative difference* [6], might be useful for quantitatively studying the limitation of the truncated Pareto distribution. It is also for our further study. We believe, however, that the heavy-tailed characteristic is *invariant* in the Internet traffic, and the truncated Pareto distribution can capture this characteristic quite well. To support our claim, some examples of parameter estimation for other packet traces are given in Appendix B.

4. Estimation of the a priori Distribution

In the preceding section, we showed that the original flow length distribution could be well modeled by the truncated Pareto distribution. In this section, we show how to estimate the unknown parameters v and θ in the truncated Pareto distribution from *sampled* flow statistics. Note that only the estimated distribution will be used as the a priori distribution in estimating the original flow length in the next section. Suppose *N* packets appear during a certain time period, and each of them is sampled independently with probability *f*. In other words, we consider random sampling from a population of *N* packets with sampling frequency *f*. We define X_j and Y_j (j = 1, 2, ..., m) as the original and sampled flow lengths of the *j*-th flow, respectively. By definition, $N = X_1 + X_2 + \cdots + X_m$. Let n_i ($i = 1, 2, ..., y_{max}$) denote the number of flows whose sampled flow length is equal to *i*, where y_{max} denotes the maximal sampled flow length. We estimate parameters θ and ν from the observed $\boldsymbol{n} = (n_1, n_2, ..., n_{y_{max}})$.

For simplicity, we assume that v can be estimated independent of θ . Recall that the ML estimator \hat{v} of v is given by $\max(X_j)$. Further the unbiased estimator of $\max(X_j)$ is given by $\max(Y_j)/f$. Therefore it is possible to estimate \hat{v} as

$$\hat{\nu} = \frac{y_{\text{max}}}{f}.$$
(3)

As we shall see in Sect. 7, the estimation error in \hat{v} does not play a crucial role in inferring the original flow length.

Due to random sampling, the conditional sampled probability $q(y | x) = \Pr[Y_j = y | X_j = x]$ is given by

$$q(y \mid x) = \begin{cases} \binom{x}{y} \cdot f^y \left(1 - f\right)^{x - y}, & y = 0, 1, \dots, x, \\ 0, & \text{otherwise.} \end{cases}$$

Thus the probability $r(y) = \Pr[Y_i = y]$ is given by

$$r(y) = \sum_{k=y}^{\nu} q(y \mid k) p(k, \theta),$$

where

$$p(k,\theta) = \Pr\left[X_j = k\right] = \frac{k^{-\theta} - (k+1)^{-\theta}}{1 - (\hat{\nu} + 1)^{-\theta}}.$$
(4)

Given the observed data $\mathbf{n} = (n_1, n_2, \dots, n_{y_{\text{max}}})$, the likelihood function with parameter θ is given by

$$L(\theta; \mathbf{n}) = \prod_{i=1}^{y_{\text{max}}} \left(\frac{r(i)}{1 - r(0)}\right)^{n_i},$$

since we cannot observe flows without sampled packets. The log likelihood function is then given by

$$\mathcal{L}(\theta) = \log L(\theta; \mathbf{n})$$

= $\sum_{i=1}^{y_{\max}} n_i \log \left(\sum_{k=i}^{\hat{v}} q(i \mid k) p(k, \theta) \right)$
 $- \left(\sum_{i=1}^{y_{\max}} n_i \right) \log \left(1 - \sum_{k=1}^{\hat{v}} q(0 \mid k) p(k, \theta) \right).$

Thus, the ML estimator $\hat{\theta}$ of parameter θ is given by a positive solution of $\partial \mathcal{L}(\theta)/\partial \theta = 0$, i.e.,



Fig. 3 Estimated a priori distributions with $f = 10^{-3}$ and $f = 10^{-4}$.

$$\begin{pmatrix} \sum_{i=1}^{\hat{y}_{\text{max}}} n_i \end{pmatrix} \frac{\sum_{k=1}^{\hat{y}} q\left(0 \mid k\right) \frac{\partial p(k,\theta)}{\partial \theta}}{1 - \sum_{k=1}^{\hat{y}} q\left(0 \mid k\right) p\left(k,\theta\right)} \\ + \sum_{i=1}^{y_{\text{max}}} n_i \frac{\sum_{k=i}^{\hat{y}} q\left(i \mid k\right) \frac{\partial p(k,\theta)}{\partial \theta}}{\sum_{k=i}^{\hat{y}} q\left(i \mid k\right) p\left(k,\theta\right)} = 0$$

where

$$\frac{\partial p(k,\theta)}{\partial \theta} = \frac{\{\log (k+1)\}(k+1)^{-\theta} - (\log k)k^{-\theta}}{1 - (\hat{v}+1)^{-\theta}} - \frac{\{\log (\hat{v}+1)\}(\hat{v}+1)^{-\theta}\{k^{-\theta} - (k+1)^{-\theta}\}}{\{1 - (\hat{v}+1)^{-\theta}\}^2}$$

We can numerically solve the equation by using a standard algorithm such as the false position method.

4.2 Evaluation of the Estimation

To evaluate the accuracy of the estimation, we first apply random packet sampling[†] to trace-I and obtain the sampled flow statistics, i.e., $\mathbf{n} = (n_1, n_2, ..., n_{y_{max}})$, for sampling frequency $f = 10^{-3}$ and $f = 10^{-4}$, respectively. Applying the MLE method described in the preceding subsection to these data, we estimate parameters \hat{v} and $\hat{\theta}$. Figure 3 shows LLCD plots of the original a priori distribution (circles) and the estimated distributions (dashed curves). For the sake of comparison, we also plot the truncated Pareto approximation obtained from original flow data (solid curve). We observe that the estimated distributions from sampled flows fit the original a priori distribution and its approximate distribution fairly well. Figure 7 shows the corresponding Q-Q plots to further validate the accuracy of the estimation. We

[†]Throughout this paper, we use "uniform probabilistic sampling," which is one of the packet sampling techniques standardized by IETF psamp WG [19]. That is, we independently sample packets with a uniform probability p = f. We also confirmed that other sampling processes such as systematic sampling give similar results.

Table 2 Estimated parameters for sampling frequency $f = 10^{-3}$ and $f = 10^{-4}$.



Fig. 4 Sampled flow statistics *n* for sampling frequency $f = 10^{-3}$ (left) and $f = 10^{-4}$ (right).

 Table 3
 Estimated parameters from 10 random sampling processes.

	min ŷ	$\max \hat{v}$	$\overline{\hat{v}}$	$\min \hat{\theta}$	$\max \hat{\theta}$	$\hat{\theta}$
$f = 10^{-3}$	225,000	266,000	244,800	.699	.716	.710
$f = 10^{-4}$	210,000	280,000	237,000	.831	.880	.851

observe that the curve is fairly close to a straight line with slope 1 over a wide range, although there are some deviations.

Table 2 shows the values of the estimated parameters \hat{v} and $\hat{\theta}$. We note that the estimated distribution becomes more heavy-tailed when f is higher, i.e., the estimated shape parameter $\hat{\theta}$ for $f = 10^{-3}$ is smaller than that of $f = 10^{-4}$. This behavior can be explained as follows. In Fig. 4, which shows the measured *n* for each sampling frequency, we observe that more sampled information comes from small *i*'s, such as $1 \le i \le 50$ for $f = 10^{-3}$ and $1 \le i \le 10$ for $f = 10^{-4}$. These *i*-packet (sampled) flows roughly correspond to $i \times 1,000$ -packet (original) flows for $f = 10^{-3}$ and $i \times 10,000$ -packet (original) flows for $f = 10^{-4}$, respectively. Thus, for $f = 10^{-4}$, the estimated flow length distribution reflects larger original flows better than the case for $f = 10^{-3}$. Since we parameterize the distribution with the truncated Pareto distribution, the estimated distribution for $f = 10^{-4}$ fits the original downward curve better, and accordingly becomes less heavy-tailed than the case for $f = 10^{-3}$.

To confirm the above explanation, we then apply packet sampling to trace-I with different random seeds for 10 times, and estimate the parameters for each sampled flow statistics. Table 3 summarizes the results. As being indicated above, the estimated $\hat{\theta}$ for $f = 10^{-4}$ is larger than that for $f = 10^{-3}$. We can also find that the estimated $\hat{\nu}$ for $f = 10^{-3}$ is relatively close to the estimated $\hat{\nu}$ for original flows without sampling (recall that $\hat{\nu} = 244,739$ in Table 1).

Thus, even though we take account of the *non-sampled* flows by introducing conditional probability, the estimated

flow length distribution is slightly skewed due to the information loss caused by packet sampling. As we shall show shortly, the difference in estimated flow length distribution does not affect the inference of the original flow length from sampled flow statistics with small false probabilities. Some examples of the estimation for other packet traces are given in Appendix B.

5. Inferring Original Flow Length

In this section, we provide a framework to infer the original flow length from the sampled flow length. Our approach is based on Bayes' theorem with the a priori distribution estimated in the preceding section.

5.1 Inference Framework

Recall that our task is to answer the following question: How many (Y) sampled packets of a specific flow should be collected to conjecture that the original flow has more than X packets in total? Let $y' = y'(x^*)$ denote the length threshold of sampled flows. Namely, if the sampled flow length of a particular flow is no less than y', we conjecture that the flow contains at least x^* packets. In what follows, we describe a way to find an appropriate value of y'.

To obtain an appropriate y', we have to consider and balance two kinds of false probabilities listed below.

$$FPR(y') \stackrel{\text{def}}{=} 1 - \Pr\left[X_j \ge x^* \mid Y_j \ge y'\right],$$

$$FNR(y') \stackrel{\text{def}}{=} 1 - \Pr\left[Y_j \ge y' \mid X_j \ge x^*\right].$$

Note that FPR(y') denotes the false positive ratio, i.e., the conditional probability that the original flow length is *less* than x^* given that the sampled flow length is not less than y'. On the other hand, FNR(y') denotes the false negative ratio, i.e., the conditional probability that the sampled flow length is *less* than y' given that the original flow length is not less than x^* .

According to Bayes' theorem, the conditional probability of $X_j \ge x^*$ given $Y_j \ge y'$ is obtained to be

$$\Pr\left[X_j \ge x^* \mid Y_j \ge y'\right]$$
$$= \frac{\sum_{k=x^*}^{\nu} \Pr\left[Y_j \ge y' \mid X_j = k\right] \Pr\left[X_j = k\right]}{\sum_{k=1}^{\nu} \Pr\left[Y_j \ge y' \mid X_j = k\right] \Pr\left[X_j = k\right]}$$

where $\Pr \left[X_j = k \right] = p(k, \hat{\theta})$ denotes the a priori distribution (see Eq. (4) in Sect. 4) and

$$\Pr\left[Y_{j} \ge y' \mid X_{j} = k\right] = 1 - \sum_{i=0}^{y'-1} \Pr\left[Y_{j} = i \mid X_{j} = k\right]$$
$$= 1 - \sum_{i=0}^{y'-1} q(i \mid k).$$

Thus, we can calculate FPR(y') by

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$$\operatorname{FPR}(y') = 1 - \frac{\sum_{k=x^*}^{\nu} \left(1 - \sum_{i=0}^{y'-1} q(i \mid k)\right) \Pr\left[X_j = k\right]}{\sum_{k=1}^{\nu} \left(1 - \sum_{i=0}^{y'-1} q(i \mid k)\right) \Pr\left[X_j = k\right]}.$$
(5)

Similarly, FNR(y') can be calculated by

$$FNR(y') = 1 - \frac{\sum_{k=x^*}^{\nu} \left(1 - \sum_{i=0}^{y'-1} q(i \mid k)\right) \Pr\left[X_j = k\right]}{\sum_{k=x^*}^{\nu} \Pr\left[X_j = k\right]}.$$
(6)

Intuitively, for a fixed x^* , FPR(y') will be a decreasing function of y', while FNR(y') will be an increasing function of y'. That is, there is an intrinsic trade-off between false probabilities. Therefore it prevents us from choosing an arbitrary y'. Our framework allows us to quantify this trade-off and make a proper choice, as we shall see shortly.

5.2 Evaluation of the Framework

To evaluate our framework, we use trace-I again (i.e., $N = 10^7$) and compare FPR(y') in Eq. (5) and FNR(y') in Eq. (6) with various a priori distributions:

- the empirical distribution of original flows,
- the truncated Pareto approximation of the empirical distribution (given in Sect. 3),
- the estimated truncated Pareto distribution from the sampled flow statistics (given in Sect. 4), and
- the uniform distribution, i.e., $\Pr[X_j = i] = 1/\max(X_j)$ $(i = 1, 2, \dots, \max(X_j)).$

The threshold x^* is set to be 10,000. FPR(y') and FNR(y') with the empirical distribution are considered as benchmarks. We plot false probabilities derived from 10 random sampling processes, and false probabilities are calculated by

$$FPR(y') = 1 - \frac{N_f (X_j \ge x^*, Y_j \ge y')}{N_f (Y_j \ge y')},$$
(7)

$$FNR(y') = 1 - \frac{N_f (X_j \ge x^*, Y_j \ge y')}{N_f (X_j \ge x^*)},$$
(8)

where $N_f(\chi)$ denotes the number of sampled flows included in event χ . We show the mean value with dashed error bar, which indicates the maximum and minimum values among those of 10 sampling processes.

In the case with the estimated truncated Pareto distribution, we use the sampled flow statistics derived from 10 random packet sampling processes for sampling frequency $f = 10^{-3}$ and $f = 10^{-4}$, respectively. Again, we plot the mean value with error bar, which indicates the maximum and minimum values. The results are shown in Figs. 5 and 6.

From these figures, we first observe that there are intrinsic trade-offs between false probabilities. In all cases, an increase in y' leads to a decrease in FPR(y') and an increase in FNR(y'). We can also see that the trade-off is more



Fig.5 False probabilities for sampling frequency $f = 10^{-3}$: FPR (y') (top) and FNR (y') (bottom). Note that the uniform distribution model falsely underestimates the false probabilities.

critical at the lower sampling frequency, i.e., $f = 10^{-4}$.

Secondly, FPR's derived from the approximate distribution (i.e., without sampling) and the estimated distributions (i.e., with sampling) fit the one obtained with the empirical distribution fairly well. Thus these FPR's seem very reliable. On the other hand, there exist relatively large discrepancies in FNR. These discrepancies are caused by the fact that the approximate distribution and the estimated distribution are slightly different from the empirical distribution in the tail quantiles (see Fig. 7). We can see that the discrepancies become larger when sampling rate is equal to 10^{-3} , where the discrepancies in the tail quantiles also become larger. Thus this might indicate the limitation of the truncated Pareto approximation. We also observe that the false probabilities derived from the estimated distributions (i.e., with or without sampling) have pretty small discrepancies; i.e., the results are still reliable.

Finally, we observe that the uniform distribution as the a priori distribution yields quite different false probabilities. The uniform distribution model assumes that the



Fig. 6 False probabilities for sampling frequency $f = 10^{-4}$: FPR (y') (top) and FNR (y') (bottom). Note that the uniform distribution model falsely underestimates the false probabilities.



Fig.7 Q-Q plot for the flow length distribution: empirical distribution vs. estimated distribution with sampling.

flow length is uniformly distributed, while the heavy-tailed model assumes that most flows have small length. Accordingly, the assumption of the uniform distribution falsely underestimates the false probabilities.

Thus, we may conclude that (i) it is essential to use an appropriate a priori distribution for inferring the original flow length from the sampled flow length, and (ii) we can successfully control FPR with the truncated Pareto distribution.

6. Identifying Heavy-Hitter Flows

As shown in the preceding section, we have to balance the trade-off between false probabilities appropriately when inferring the original flow length from the sampled flow length. In this section, we present an example of identifying heavy-hitter flows in order to demonstrate such a balancing.

To balance the trade-off between false probabilities, we consider the following policy as a guideline: For a given sampling frequency, the false positive ratio should be less than a predefined threshold ϵ , while the false negative ratio should remain reasonably low. Note that this policy is useful in the following situation. Assume that an ISP wants to regulate heavy-user's traffic when a link is severely congested. For this purpose, the ISP is monitoring traffic flows that traverse its network with packet sampling, and it wants to identify heavy-hitter flows quickly from sampled flow statistics. Qualitatively, the characterization of heavy-hitter flows is that they occupy the majority of total traffic volume (in terms of the number of packets or bytes). Note that, due to heavytailed characteristics, the number of heavy-hitter flows is very small, compared with the total number of flows. Therefore it is quite effective to regulate their transmission rate when the link is congested. The quantitative definition of a heavy-hitter flow can be arbitrarily determined by network operators according to their own criterion. It may be possible that the criterion is used for the agreement between the ISP and their users, where too much false positives are not preferred for the ISP.

As an example, we define a heavy-hitter flow as a flow with $X_j \ge 10,000$; i.e., we set x^* to be $10,000^{\dagger}$. According to this definition, the number of heavy-hitter flows in trace-I is equal to 170, while the total number of flows is 206,299. Note that the heavy-hitter flows contribute about 53% of the total traffic volume in bytes, whereas they contribute only .084% in terms of the number of flows. Based on these observations, our policy seems to be useful for the following reasons. First, we can avoid mistreating *non-heavy-hitter* flows (e.g., shaping their packet rate) when the false positive ratio is sufficiently low. Secondly, even if a low false positive ratio may cause a relatively high false negative rate,

[†]Although we define a heavy-hitter flow by their number of packets (not by their bytes), the information of the heavy-hitter flows by this definition is quite useful, e.g., identifying severe network attacks, misconfiguration, and another heavy data streams in the opposite direction of the link (i.e., the identified flows might be ack streams).

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Table 4

ŷ	$FPR(\hat{y})$	$FNR(\hat{y})$	
13	.039	.178	
у	real $FPR(y)$	real FNR(y)	
<i>y</i> 13	real FPR(y) .057	real FNR(<i>y</i>) .318	

 \hat{y} , FPR(\hat{y}) and FNR(\hat{y}) for trace-I ($\epsilon = .05$, $f = 10^{-3}$).

Table 5 \hat{y} , FPR(\hat{y}) and FNR(\hat{y}) for trace-I ($\epsilon = .05$, $f = 10^{-4}$).

ŷ	$FPR(\hat{y})$	$FNR(\hat{y})$	
4	.020	.667	
у	real $FPR(y)$	real $FNR(y)$	
3	.075	.712	

the amount of traffic volume generated by identified heavyhitter flows is already significant. For example, the 10 heaviest flows in trace-I account for about 17% of the total traffic volume.

According to this policy, our goal is to find $y' = \hat{y}$ such that FPR(\hat{y}) is no greater than ϵ . Because FPR(\hat{y}) is a decreasing function of y', threshold \hat{y} can be determined by

$$\hat{y} = \min_{y'} \left\{ y' \mid \text{FPR}(y') \le \epsilon \right\}.$$
(9)

Note that Eq. (9) guarantees the lowest false negative ratio under the constraint $FPR(y') \le \epsilon$, since FNR(y') is an increasing function of y'.

To evaluate this policy, we apply a random sampling process to trace-I, and derive sampled flow statistics. Here, the sampling frequencies are set to be $f = 10^{-3}$ and $f = 10^{-4}$. We then estimate the flow length distribution from the sampled flow statistics. Using the estimated flow length distributions and Eq. (9), we calculate \hat{y} , FPR(\hat{y}), and FNR(\hat{y}) for each sampling frequency. Tables 4 and 5 list these results, where ϵ is set to be .05. We also show the benchmarks of these values, which are obtained through Eqs. (7) and (8), for the sake of comparison with the results from the sampled flow statistics.

When $f = 10^{-3}$, \hat{y} is estimated to be 13 with FPR(13) = .039. On the other hand, in this specific sampled data, the real FPR for y = 13 is .057 and the real \hat{y} is 14. Thus there exists an estimation error. As we observed in the Sect. 5, the estimated FNR is smaller than the real value. When $f = 10^{-4}$, the estimated \hat{y} is the same as the real value and the corresponding FPR(\hat{y}) = .020 is more conservative, i.e., it is greater than the real value 0. Apparently this result is preferable according to our criteria. We observe, however, that FNR is underestimated again.

Before closing this section, we summarize the procedure for identifying heavy-hitter flows from sampled flow statistics. We also give a guideline to set the initial parameters in Step 1.

Step 1: Determine (i) sampling frequency f, (ii) threshold x^* defining a heavy-hitter flow, and (iii) threshold ϵ that FPR should satisfy.

- **Step 2:** Estimate an *a priori* distribution of X_j from sampled flow statistics *n* according to Sect. 4.
- **Step 3:** Calculate \hat{y} according to Eq. (9).
- **Step 4:** If the sampled flow length of a flow is not less than \hat{y} , the flow is identified as a heavy-hitter flow.

Guideline of setting initial parameters

The initial parameters in Step 1 can be determined by network operators according to their policies and available resources. However, these parameters are not totally independent and there is a trade-off between cost and accuracy. As we have shown above, our framework is able to address this trade-off quantitatively, which makes possible for network operators to choose these parameters flexibly and effectively. First, the network operators might want to set x^* when they try to regulate traffic from heavy hitter flows whose size exceeds certain threshold, x^* . Then, when they set the sampling frequency f, they can refer to the performance of their network equipment. Through the discussion with network operators, we have found that in most cases, they are using *rule-of-thumb* when they set the sampling frequency, e.g., $f = 10^{-3}$, 10^{-4} , etc, which we chose as examples in this paper. However, we believe that it is possible to set the sampling frequency more flexibly rather than using the rule-of-thumb as we will show below. Finally, the operators might want to set ϵ to be as small as possible in many cases, however, it cannot be too small, since there is an instrinsic trade-off between FPR and FNR as shown in Fig. 6 and Fig. 7. One can set ϵ to be small so that FNR does not increase too much. If one prefers both FPR and FNR to be small, it can be achieved by increasing the sampling frequency as much as their resource allows.

7. Further Discussion

Influence of \hat{v} — In Sect. 4.1, we adopt $\hat{v} = y_{\text{max}}/f$ as an estimator of parameter v (see Eq. (3)). Here we discuss how the estimation error in \hat{v} affects our scheme.

Consider the situation where the *j*-th flow consists of X_j packets and Y_j packets are sampled with sampling frequency *f*. As shown in [5], $\hat{X}_j = Y_j/f$ is an unbiased estimator of X_j and the standard deviation $\sigma_{\hat{X}_j}$ of \hat{X}_j are bounded above by $\sqrt{X_j/f}$. To see how the estimation error in \hat{X}_j affects our scheme, we consider the following v_1^* and v_2^* .

$$v_1^* = \max X_j - 2\sqrt{\max X_j/f},$$

$$v_2^* = \max X_j + 2\sqrt{\max X_j/f}.$$

Roughly speaking, the estimated $\hat{v} = y_{\text{max}}/f$ lies in $[v_1^*, v_2^*]$ with probability .95.

From the sampled flow statistics of trace-I used in Sect. 4.2, we obtain two estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ of parameters

Table 6 Estimated $\hat{\theta}_1$ and $\hat{\theta}_2$ for v_1^* and v_2^* .





Fig. 8 False probabilities for sampling frequency $f = 10^{-3}$ (top) and $f = 10^{-4}$ (bottom).

 θ using v_1^* and v_2^* , respectively. Table 6 summarizes the results. Recall that max $(X_j) = 244,739$ for trace-I. We can observe that the estimated $\hat{\theta}_1$ and $\hat{\theta}_2$ are quite close to each other. Thus, we could confirm that the estimation error of \hat{v} does not strongly affect the estimation of θ .

Next, we examine how the estimation error in \hat{v} affects our framework proposed in Sect. 5.1. Since FPR(y') in Eq. (5) and FNR(y') in Eq. (6) are based on the a priori distribution $\Pr[X_j = x]$, it is possible that the estimation error in \hat{v} affects these false probabilities. We use the truncated Pareto distributions with sets of parameters ($v_1^*, \hat{\theta}_1$) and ($v_2^*, \hat{\theta}_2$) in Table 6 as a priori distributions, and calculate false probabilities for sampling frequency $f = 10^{-3}$ and $f = 10^{-4}$. Figure 8 shows the results. We observe that the false probabilities

bilities in both cases are almost indistinguishable, especially for FPR. Thus, we may conclude that the estimation of \hat{v} by Eq. (3) is reasonable for our scheme.

8. Conclusions

In this paper, we developed techniques and schemes to evaluate the original flow length from the sampled flows. Our approach consists of two stages. The first stage is to conduct a parametric estimation of the original flow length distribution from sampled flows. The second stage is to use Bayes' theorem with the estimated a priori distribution for inferring the original flow length from the sampled flow length. We showed that our basic framework is very flexible in striking an appropriate balance between false positive and false negative when sampling frequency was given. We further validated and evaluated our approach with some publicly available traces. Our schemes are generic and require *no* per-packet processing; hence, they allow a very cost-effective implementation for being deployed in largescale high-speed networks.

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Appendix A: Parameter Estimation of the Truncated Pareto Distribution

This section proves Eqs. (1) and (2), which are ML estimators of parameters in the truncated Pareto distribution. Given the observed flow lengths $X_1, X_2, ..., X_m$, the likelihood function $L(\nu, \theta)$ is given by

$$L(\nu,\theta) = \prod_{j=1}^{m} \frac{X_{j}^{-\theta} - (X_{j}+1)^{-\theta}}{1 - (\nu+1)^{-\theta}}.$$

Because $L(\nu, \theta)$ is a decreasing function of ν and $X_j \le \nu < \infty$, we obtain $\nu = \hat{\nu}$, where $\hat{\nu}$ is given in Eq. (1).

Note here that the log likelihood function $\mathcal{L}(\theta)$ is given by

$$\mathcal{L}(\theta) = -m \log \left\{ 1 - (\nu+1)^{-\theta} \right\} + \sum_{j} \log X_{j}^{-\theta} - \left(X_{j} + 1 \right)^{-\theta}.$$

Thus, $\theta = \hat{\theta}$, which maximizes the log likelihood function, satisfies the following equation

$$\frac{\partial \mathcal{L}\left(\theta\right)}{\partial \theta} = 0,$$

which yields Eq. (2).

Appendix B: Examples of Parameter Estimation

To supplement the results obtained from trace-I, we used other packet traces as well. One was measured at the same link of trace-I but during a different time period: from 10:00 to 10:10 on the same day. We use the first 10^7 packets that correspond to about 138 seconds of the observed traffic. We call this sliced trace as trace-II, which contains 197,536 flows. The other was measured at an OC-192 backbone link [18] from 23:00 to 23:10 on June 1, 2004. Again, we use the first 10^7 packets that correspond to about 99 seconds of observed traffic. We call this sliced trace trace-III, which contains 498,036 flows.



Fig. A·1 The estimated distributions from sampled flow statistics for trace-II (top) and trace-III (bottom) $(f = 10^{-3}, 10^{-4})$.

First, we approximate the original flow length distributions by the truncated Pareto distribution. We then apply random packet sampling to the trace-II and trace-III, with sampling frequency of $f = 10^{-3}$ and $f = 10^{-4}$. From the obtained sampled flow statistics \mathbf{n} , we estimate parameters v and θ of the truncated Pareto distribution. Figure A·1 shows the results. We can observe that the approximate distribution and the estimated distribution visually fit well to the empirical distributions. Although the approximate and the estimated distributions, we confirmed that the difference did not play a crucial role in determining the thresholds of sampled flow lengths, which identify the heavy-hitter flows.

Since our parametric model assumes that the flow length distribution follows the truncated Pareto distribution, it would work well as far as the original flow distributions follow the heavy-tailed model. We believe that this assumption holds in most cases as reported in many measurementbased studies such as [9], [14], [20], [22], [23]. Moreover, the reason why the heavy-hitter flows exist among flows comes from the fact that the distribution is heavy-tailed. However, it might be possible that the original flow length distribution shows quite different characteristics. For such distributions, we might want to introduce another type of distribution model in our framework.



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